# Rare Semileptonic Decays of Heavy Mesons with Flavor SU(3) Symmetry

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In this paper, we calculate the decay rates of $D^+ \to D^0 e^+ \nu$, $D_S^+ \to D^0 e^+ \nu$, $B_S^0 \to B^+ e^- \overline{\nu}$, $D_S^+ \to D^+ e^- e^+$ and $B_S^0 \to B^0 e^- e^+$ semileptonic decay processes, in which only the light quarks decay, while the heavy flavors remain unchanged. The branching ratios of these decay processes are calculated with the flavor SU(3) symmetry. The uncertainties are estimated by considering the SU(3) breaking effect. We find that the decay rates are very tiny in the framework of the Standard Model. We also estimate the sensitivities of the measurements of these rare decays at the future experiments, such as BES-III, super-$B$ and LHC-$b$.

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The properties of hadrons containing a single heavy quark Q ($m_Q \gg \Lambda_{QCD}$) along with light degrees of freedom are constrained by symmetries which are not apparent in QCD [1]. These symmetries are manifest in the heavy quark effective theory (HQET) where the heavy quark acts in the hadron's rest frame like a spatially static triplet source of color electric field [2, 3]. In the HQET the heavy quark's coupling to the gluon degrees of freedom are independent of its mass and described by a Wilson line [4]. In the rare decay processes of $D^+ \to D^0 e^+ \nu$, $D_S^+ \to D^0 e^+ \nu$, $B_S^0 \to B^+ e^- \overline{\nu}$, $D_S^+ \to D^+ e^- e^+$ and $B_S^0 \to B^0 e^- e^+$, the heavy quark flavors ($c$ or $b$) remain unchanged, and the weak decays are managed by the light quark sectors. In the limit of the flavor SU(3) symmetry of the light quarks, the matrix elements of the weak current can be constrained, the uncertainty can be estimated.

In this work, we study the rare decay processes $D^+ \to D^0 e^+ \nu$, $D_S^+ \to D^0 e^+ \nu$, $B_S^0 \to B^+ e^- \overline{\nu}$, $D_S^+ \to D^+ e^- e^+$ and $B_S^0 \to B^0 e^- e^+$. Applying the SU(3) symmetry for the light quarks, the form factors describing the strong interaction in these decays can be obtained. The uncertainties can be estimated by considering SU(3) breaking effect.

For the semileptonic decays $D^+ \to D^0 e^+ \nu$ and $D_S^+ \to D^0 e^+ \nu$, the decay amplitude is

$$\begin{aligned} \mathcal{A} = \quad & \frac{G_F}{\sqrt{2}} V_{ij} \Big[ \bar{u}(k_1)\gamma^\mu (1-\gamma_5) v(k_2) \\ & \langle D^0(p_2) | \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2 | D^+_{(S)}(p_1) \rangle \Big], \qquad (1) \end{aligned}$$

where $G_F = 1.16639 \times 10^{-5}\,\mathrm{GeV}^{-2}$ is the Fermi constant, and $V_{ij}$ is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element, the functions $\bar{u}(k_1)$ and $v(k_2)$ are Dirac spinors, which describe the productions of the neutrino with momentum $k_1$ and the anti-lepton with momentum $k_2$, respectively.

According to its Lorentz structure, the hadronic matrix element in eq.(1) can be decomposed as

$$\begin{aligned} & \langle D^0(p_2) | \bar{q}_1 \gamma_\mu (1-\gamma_5) q_2 | D^+_{(S)}(p_1) \rangle \\ & = f_+(q^2)(p_1+p_2)_\mu + f_-(q^2)(p_1-p_2)_\mu, \qquad (2) \end{aligned}$$

where $f_\pm(q^2)$ are the form factors including all the dynamics of strong interaction. Considering the decomposition of the hadronic matrix element in eq.(2), and neglecting the lepton mass, we get the amplitude as

$$\mathcal{A} = \frac{G_F}{\sqrt{2}} V_{ij} f_+(q^2)(p_1+p_2)_\mu \bar{u}(k_1)\gamma^\mu(1-\gamma_5)v(k_2). \quad (3)$$

Then the decay width can be obtained as

$$\begin{aligned} \Gamma \;=\; & \frac{1}{192\pi^3 m_1^3} G_F^2 |V_{ij}|^2 \times \\ & \int dq^2 f_+^2(q^2) \left[ (m_1^2+m_2^2-q^2)^2 - 4m_1^2 m_2^2 \right]^{3/2}, (4) \end{aligned}$$

where $m_1$ and $m_2$ are the masses of the initial and final heavy hadron involved in these rare decays.

Next we consider the flavor-changing neutral current (FCNC) processes $D_S^+ \to D^+ e^- e^+$ and $B_S^0 \to B^0 e^- e^+$. They can be described by the $\Delta S = 1$ effective Hamiltonian in the quark level at scales $\mu < m_c$ [5]:

$$\begin{aligned} \mathcal{H}_{eff}^{\Delta S=1} \;=\; & \frac{G_F}{\sqrt{2}} \Big[ \sum_{i=1}^{6,7V} (V_{us}^* V_{ud} Z_i(\mu) - V_{ts}^* V_{td} Y_i(\mu)) Q_i(\mu) \\ & - V_{ts}^* V_{td} Y_{7A}(m_W) Q_{7A}(m_W) \Big], \qquad (5) \end{aligned}$$

where the operators $Q_i(\mu)$'s are defined as

$$\begin{aligned}
Q_1 &= (\bar{s}_\alpha u_\beta)_{V-A}(\bar{u}_\beta d_\alpha)_{V-A} \\
Q_2 &= (\bar{s}u)_{V-A}(\bar{u}d)_{V-A} \\
Q_3 &= (\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V-A} \\
Q_4 &= (\bar{s}_\alpha d_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A} \\
Q_5 &= (\bar{s}d)_{V-A}\sum_q(\bar{q}q)_{V+A} \\
Q_6 &= (\bar{s}_\alpha d_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V+A} \\
Q_7 &= (\bar{s}d)_{V-A}(\bar{e}e)_V \\
Q_{7A} &= (\bar{s}d)_{V-A}(\bar{e}e)_A. \qquad (6)
\end{aligned}$$

Here the indexes $\alpha$ and $\beta$ are color numbers, the summation $\sum$ runs over all the quark flavors which are active at the scale $\mu$, and the $V \pm A$ denotes $\gamma_\mu(1 \pm \gamma_5)$.

At $\mu = 1$ GeV with $\Lambda^{(4)}_{\overline{\mu s}} = 215$ MeV and NDR scheme, the Wilson coefficients $Z_i$ and $Y_i$'s are calculated to be [5] $Z_1 = -0.409$, $Z_2 = 1.212$, $Z_3 = 0.008$, $Z_4 = -0.022$, $Z_5 = 0.006$, $Z_6 = -0.022$, $Z_{7V} = -0.015\alpha_{QED}$, and $Y_1 = Y_2 = 0$, $Y_3 = 0.025$, $Y_4 = -0.048$, $Y_5 = 0.005$, $Y_6 = -0.078$, $Y_{7V} = 0.747\alpha_{QED}$, $Y_{7A} = -0.700\alpha_{QED}$, where $\alpha_{QED}$ is the fine structure constant.

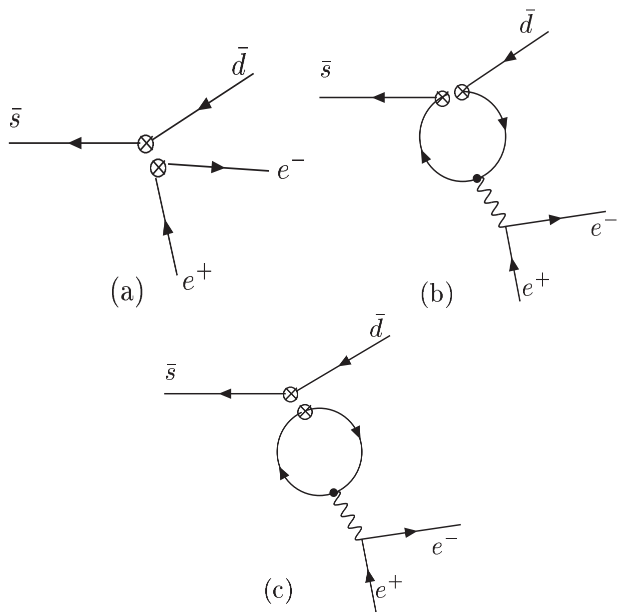


FIG. 1: Diagrams for the rare decays of $D^+_S \to D^+e^+e^-$ and $\bar{B}^0_S \to \bar{B}^0e^+e^-$ up to one-loop level, where the cross circles denote the operator insertion. (a): the tree level diagram; (b) and (c): the one-loop level diagrams with two different ways of operator insertions.

We calculate the FCNC processes $D^+_S \to D^+e^+e^-$ and $\bar{B}^0_S \to B^0e^+e^-$ up to one-loop level based on the effective Hamiltonian. The diagrams we considered are depicted in Fig.1. The amplitude for the FCNC processes is calculated to be

$$\begin{aligned}
\mathcal{A} &= \left[a_1\bar{u}(k_1)\gamma^\mu v(k_2) + a_2\bar{u}(k_1)\gamma^\mu\gamma_5 v(k_2)\right] \\
&\quad \times(p_1+p_2)_\mu f_+(q^2), \qquad (7)
\end{aligned}$$

with

$$\begin{aligned}
a_1 &= \frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}\Big\{Z_{7V} + \tau Y_{7V} + \frac{N_c\alpha_{QED}}{2\pi}\times \\
&\Big[\big((Z_1+\frac{Z_2}{N_c})Q_u + (\frac{Z_3}{N_c}+Z_4)Q_d + \tau(\frac{Y_3}{N_c}+Y_4)Q_d\big)\times \\
&\big(-\frac{2}{3}+G(0,q^2,\mu)\big) + \big(Z_3+\frac{Z_4}{N_c}+Z_5+\frac{Z_6}{Nc}+ \\
&\tau(Y_3+\frac{Y_4}{N_c}+Y_5+\frac{Y_6}{N_c})\big)\big((Q_u+Q_d)G(0,q^2,\mu)+ \\
&Q_sG(m_s^2,q^2,\mu)\big) + (Z_{7V}+\tau Y_{7V})\frac{Q_e}{N_c}G(0,q^2,\mu)\Big]\Big\}, \qquad (8) \\
a_2 &= \frac{G_F}{\sqrt{2}}V^*_{us}V_{ud}\tau Y_{7A}, \qquad (9)
\end{aligned}$$

where $N_c = 3$ is the color number, $Q_u$, $Q_d$, $Q_s$ and $Q_e$ are the charge of the relevant quarks and electrons, $m_s$ is the mass of the strange quark, and the function $G$ is defined as

$$\begin{aligned}
G(m^2,q^2,\mu) &= -\int_0^1 dx 4x(1-x)\times \\
&\quad \ln\frac{m^2-x(1-x)q^2-i\epsilon}{\mu^2}, \qquad (10)
\end{aligned}$$

which is originated from the loop calculation. The parameter $\tau$ is defined to be

$$\tau = -\frac{V^*_{ts}V_{td}}{V^*_{us}V_{ud}}. \qquad (11)$$

With the amplitude in eq.(7), the decay width can be obtained as

$$\begin{aligned}
\Gamma &= \frac{1}{192\pi^3m_1^3}\int dq^2 f_+(q^2)^2(|a_1|^2+|a_2|^2)\times \\
&\quad \left[(m_1^2+m_2^2-q^2)^2-4m_1^2m_2^2\right]^{3/2}. \qquad (12)
\end{aligned}$$

In the numerical calculations we use the following values for the Wolfenstein parameters [6]:

$$\lambda = 0.2272;\ A = 0.818;\ \bar{\rho} = 0.221;\ \bar{\eta} = 0.340. \qquad (13)$$

In the limit of SU(3) symmetry for the light quarks $u$, $d$ and $s$, the masses of the light quarks are the same, the masses of the mesons with the same heavy quark but different light flavors are also the same. Therefore the energy release in the rare decay process is zero in the limit of the SU(3) symmetry. The velocity of the heavy mesons in the initial and final states is unchanged. The wave functions of the light quarks are the same after the rare decay occurs. The heavy quark does not feel any

change during the light flavor transition process. Thus with the flavor SU(3) symmetry holding, the form factor $f_+(q^2)$ can be normalized to unity at the point $q^2 = 0$,

$$f_+(0) = 1. \tag{14}$$

Note that there is only one kinematic value for $q^2$ with the SU(3) symmetry holding, $q^2 = 0$. However, with SU(3) flavor symmetry broken, i.e., the mass of $s$ quark $m_s$ being larger than that of $u$ and $d$ quarks, the light quark is boosted after the light flavor transition occurs. The possibility for the light and heavy quarks still bound together to form a heavy meson decreases, this leads to a form-factor suppression. Thus the value of the form factors must be smaller than unity. The larger the $s$ quark mass, the larger the deviation.

With the SU(3) symmetry broken, the allowed value of $q^2$ is extended from $q^2 = 0$ to a range of $0 \leq q^2 \leq q^2_{max}$. At the point $q^2 = q^2_{max}$, all the energy and momentum released in the decay are carried away by the lepton pair, the velocity of the final meson is not changed (this is the point of zero recoil) , only the mass of the light quark changed, therefore the deviation from the limit of SU(3) symmetry is the smallest at the point of zero recoil. For the point $q^2 = 0$, there is further deviation caused by the kinematic change of the final meson. Thus one can write the the form factor at $q^2 = q^2_{max}$ as an expansion in terms of a small SU(3) breaking parameter. Including the SU(3) breaking effect, eq.(14) is extended to the point at $q^2 = q^2_{max}$

$$f_+(q^2_{max}) = 1 + \lambda_{SU(3)}, \tag{15}$$

where the parameter $\lambda_{SU(3)}$ describes the correction due to SU(3) breaking effect at $q^2 = q^2_{max}$. Note that the value of $\lambda_{SU(3)}$ must be negative, because the SU(3) breaking effect leads to a form-factor suppression. In addition, if the expansion including the SU(3) breaking effect is performed at the point $q^2 = 0$ instead of the point of zero recoil, further correction due to the velocity change of the final meson should be considered.

The larger the mass of the $s$ quark (here we neglect the masses of $u$ and $d$ quarks), the larger the SU(3) breaking effects. An appropriate SU(3) breaking parameter can be taken to be $m_s/\Lambda$, where $\Lambda \sim$ 1GeV is a hadronic scale. According to the Ademollo-Gatto theorem [7, 8], the parameter $\lambda_{SU(3)}$ should be of the second order of the SU(3) breaking parameter, i.e.,

$$\lambda_{SU(3)} \sim \mathcal{O}((m_s/\Lambda)^2). \tag{16}$$

In this work the correction to the form factor due to the SU(3) breaking effect is treated as an uncertainty, the form factor is varied in the range

$$1 - (m_s/\Lambda)^2 \leq f_+(q^2_{max}) \leq 1. \tag{17}$$

The momentum transfer squared $q^2$ should be in the range range $0 < q^2 < (m_{H_1} - m_{H_2})^2$, where $m_{H_1}$ and $m_{H_2}$ are the masses of the initial and final mesons containing the heavy quark, respectively. The $q^2$ dependence of the form factor is typically governed by the nearest resonance, which can be parameterized as a pole-dominance form [9]

$$f_+(q^2) = \frac{f_+(0)}{1 - q^2/m^2_{H^*}}, \tag{18}$$

where $H^*$ is nearest pole resonance which contributes to the from factor. $H^*$ should be $K^*$ in $s \to d$, or $u$ transitions. With eqs.(17) and (18), we can obtain that the form factor at $q^2 = 0$ should be varied in the range

$$(1-(m_s/\Lambda)^2)(1-q^2_{max}/m^2_{H^*}) \leq f_+(0) \leq (1-q^2_{max}/m^2_{H^*}). \tag{19}$$

Taking the mass of strange quark $m_s = 100$ MeV, by considering the SU(3) breaking effect, we get the following branching fractions:

$$\begin{aligned}
\mathcal{B}(D^+ \to D^0 e^+ \nu) &= 2.78 \times 10^{-13}, \\
\mathcal{B}(D^+_S \to D^0 e^+ \nu) &= (2.97 \pm 0.03) \times 10^{-8}, \\
\mathcal{B}(B^0_S \to B^+ e^- \bar{\nu}) &= (4.46 \pm 0.05) \times 10^{-8}, \\
\mathcal{B}(D^+_S \to D^+ e^+ e^-) &= (3.56 \pm 0.04) \times 10^{-17}, \\
\mathcal{B}(B^0_S \to B^0 e^+ e^-) &= (6.44 \pm 0.07) \times 10^{-17},
\end{aligned} \tag{20}$$

where the uncertainties are caused by SU(3) breaking effect. For $D^+ \to D^0 e^+ \nu$, the correction to the form factor due to isospin breaking effect is neglected in the above presentation, the numerical correction due to isospin breaking should be far less than the effects in the strangeness-violating processes. Here we take $m_c = 1.5$ GeV, $m_b = 4.8$ GeV in the numerical calculations.

The mass of the strange quark is very close to the mass differences of the initial and final mesons in the decay processes involving $s$ quark, $D^+_S \to D^0 e^+ \nu$, $D^+_S \to D^+ e^+ e^-$, $B^0_S \to B^+ e^- \bar{\nu}$, $B^0_S \to B^0 e^+ e^-$, which is about 100 MeV. It is interesting to study the role of $m_s$ in these decay processes by slightly varying the mass of strange quark. The results are given in Table I. The decay rates are only slightly changed with $m_s$ varying in the range 100 MeV $< m_s <$ 140 MeV.

Within the SM framework, we find that the branching fractions for these rare decays are tiny. However, in the coming experiments at BES-III [10], LHC-$b$ [11] and super-$B$ factory[12], it is interesting to search for these semileptonic decays. Especially, the decays of $D^+_S \to D^0 e^+ \nu$ and $B^0_S \to B^+ e^- \bar{\nu}$ may be reached at the super-$B$ factory.

Since the electron is very soft, one cannot reconstruct both the electron and neutrino in the experiment near the charm meson threshold at $e^+e^-$ colliders. At BES-III, to search for the decay $D^+ \to D^0 e^+ \nu$ on the $\psi(3770)$ peak, the charged $D$ mesons are produced in pairs, $e^+e^- \to \psi(3770) \to D^+ D^-$. Thus, the following six tag modes, $D^- \to K^+\pi^-\pi^-$, $K^+\pi^-\pi^-\pi^0$, $K_S\pi^-$,

TABLE I: The decay rates of the rare processes involving strange quark. The second uncertainty is estimated from SU(3) breaking effect due to the mass of the strange quark.

| | $m_s = 100$MeV | $m_s = 120$MeV | $m_s = 140$MeV |
|---|---|---|---|
| $D_S^+ \to D^0 e^+ \nu$ | $(2.97 \pm 0.03) \times 10^{-8}$ | $(2.96 \pm 0.05) \times 10^{-8}$ | $(2.94 \pm 0.06) \times 10^{-8}$ |
| $B_S^0 \to B^+ e^- \bar{\nu}$ | $(4.46 \pm 0.05) \times 10^{-8}$ | $(4.43 \pm 0.07) \times 10^{-8}$ | $(4.41 \pm 0.09) \times 10^{-8}$ |
| $D_S^+ \to D^+ e^+ e^-$ | $(3.56 \pm 0.04) \times 10^{-17}$ | $(3.64 \pm 0.06) \times 10^{-17}$ | $(3.63 \pm 0.07) \times 10^{-17}$ |
| $B_S^0 \to B^0 e^+ e^-$ | $(6.44 \pm 0.07) \times 10^{-17}$ | $(6.40 \pm 0.10) \times 10^{-17}$ | $(6.37 \pm 0.13) \times 10^{-17}$ |

$K_S\pi^-\pi^-\pi^+$, $K_S\pi^-\pi^0$ and $K^+K^-\pi^-$, can be used to fully reconstruct one of the charged $D$ mesons. The summed branching fractions of the six tag modes are about 28% of all the charged $D$ decays [6]. The tag efficiency for the charged $D$ mesons are about 20%, which means that 20% of all the $D^+D^-$ pairs can be tagged [10]. For this case, in order to detect the decay $D^+ \to D^0 e^+ \nu$, one can reconstruct the neutral $D$ meson decay by using 46% of all of the $D^0$ decays modes in the tagged charged $D$ sample [10]. If we see any event of the production of the neutral $D$ mesons against the charged $D$ mesons, it indicates the observation of the rare semileptonic decay. For the decay $D_S^+ \to D^0 e^+ \nu$, the same method can be applied by using the data collected at the center of mass $E_{CM} = 4170$ MeV. With 20 fb$^{-1}$ data on the $\psi(3770)$ peak, the sensitivity of the measurement of $D^+ \to D^0 e^+ \nu$ can be $10^{-6}$ at the BES-III experiment, while, for the measurement of $D_S^+ \to D^0 e^+ \nu$, it can reach $10^{-5}$ level with 20 fb$^{-1}$ data running at $E_{CM} = 4170$ MeV. These estimations are listed in table II,

These semi-leptonic decays can also be searched in the $B$ and super-$B$ factories by using data at $\Upsilon(4S)$ peak. One can reconstruct the following decay chain to search for the rare decays $D^+ \to D^0 e^+ \nu$:

$$D^{*+} \to D^+ \pi^0_{soft}, \;\; D^+ \to D^0 e^+_{soft} \nu, \tag{21}$$

where the $D^{*+}$ is boosted, and both $\pi^0$ and electron could have momentum with a few hundred MeV, which can be detected and reconstructed in the detector. Since the missing neutrino has very low momentum, one can partially reconstruct the decay of $D^+$, and looking at the mass difference $\Delta m = m((D^0 e^+)\pi^0_{soft}) - m(D^0 e^+)$. The signal events should peak around the mass difference of $m_{D^{*+}} - m_{D^+} = 140$ MeV on the $\Delta m$ distribution. In the mass difference, the uncertainty of the reconstruction of $(D^0 e^+)$ can cancel. The resolution on the $\Delta m$ will be dominated by the detection of the soft $\pi^0$. This is a powerful variable to separate background from the signal events. For the decay $D_S^+ \to D^0 e^+ \nu$, one can use the reaction:

$$D_S^{*+} \to D_S^+ \gamma_{soft}, \;\; D_S^+ \to D^0 e^+_{soft} \nu, \tag{22}$$

to extract the rare decay signal by looking at the mass difference $\Delta m = m((D^0 e^+)\gamma_{soft}) - m((D^0 e^+)$. With 1 ab$^{-1}$ and 50 ab$^{-1}$ luminosity at $B$ factories and super-$B$, the sensitivities could be $10^{-8}$ and $10^{-10}$ respectively. In table II, the sensitiveities of the measurements of the rare $D^+$ and $D_S^+$ decays are summarized at different experiments.

At super-$B$ factory, the data taken at $\Upsilon(5S)$ can be used to search for the rare decays $B_S^0 \to B^+ e^- \bar{\nu}$ and $B_S^0 \to B^0 e^- e^+$. The cross section of the $\Upsilon(5S)$ production at $e^+e^-$ collisions is $\sigma(e^+e^- \to \Upsilon(5S)) = 0.301 \pm 0.002 \pm 0.039$ nb [13]. Unlike the $\Upsilon(4S)$ state, $\Upsilon(5S)$ is heavy enough to decay into several $B$ meson states, in which the vector-vector ($B_S^* \bar{B}_S^*$) and vector-pseudoscalar ($B_S^* \bar{B}_S + B_S \bar{B}_S^*$) combinations are dominant [14]. About 30% of the $\Upsilon(5S)$ decays into $B_S$ final states [12]. With 30 ab$^{-1}$ data at $\Upsilon(5S)$ peak at super-$B$, the sensitivity of the measurements of the rare $B_S$ decays can be $10^{-9}$ by assuming 30% efficiency for the $B_S$ reconstruction.

TABLE II: Experimental sensitivities at BES-III, $B$ factory, Super-$B$ and LHC-$b$ for the rare decays. We assume the integrated luminosities are 20 fb$^{-1}$ (BES-III at $\psi(3770)$ peak and 4170 MeV), 1 ab$^{-1}$ and 50 ab$^{-1}$ at $B$ factory and Super-$B$ (at $\Upsilon(4S)$ peak), 10 fb$^{-1}$ at LHC-$b$, respectively.

| Decays | BES-III ($\times 10^{-6}$) | $B$ factory ($\times 10^{-8}$) | Super-$B$ ($\times 10^{-10}$) | LHC-$b$ ($\times 10^{-9}$) |
|---|---|---|---|---|
| $D^+ \to D^0 e^+ \nu$ | 1.0 | 1.1 | 2.3 | 3.8 |
| $D_S^+ \to D^0 e^+ \nu$ | 5.0 | 1.1 | 2.3 | 3.8 |
| $D_S^+ \to D^+ e^+ e^-$ | 11.5 | 2.0 | 4.6 | 5.0 |

All of these rare decays can be studied at the LHC-$b$, in which production cross section for $b\bar{b}$ is 500 $\mu b$. About $5 \times 10^{12}$ $b\bar{b}$ pairs will be obtained in 10 fb$^{-1}$ integrated luminosity in the first five years running at the LHC-$b$ [11]. Accordingly, about 40% (40% or 10%) of the $b\bar{b}$ pairs is predicted to form $B^0$ meson ($B^+$ or $B_S^0$ meson). Thus, the sensitivity of the measurements of the rare $B_S$ decays is estimated to be $10^{-10}$ at the LHC-$b$ with 10 fb$^{-1}$ luminosity. Both the $D^{*+}$ and $D_S^{*+}$ can be reconstructed in the decays of $B$ and $B_S^0$ mesons, the decay chains in Eqs. 21 and 22 can also be used to extract the rare decay signals of $D^+$ and $D_S^+$ mesons. The estimated sensitivities for the rare decays of the charm mesons are about $10^{-9}$ at the LHC-$b$. However, one has to be careful that there are many unexpected backgrounds at the LHC-$b$ experiment. The estimated sensitivity should be more conservative.

In summary, for the first time, we calculated the decay rates of the rare $D^+$, $D_S^+$ and $B_S$ decays, in which only the light quarks decay weakly, while the heavy fla-

vors remain unchanged. Applying the SU(3) flavor symmetry, the form factors describing the strong interaction in these decays can be obtained. Considering SU(3) symmetry breaking, the uncertainty for the form factors can be estimated. Therefore, these rare decays can be predicted with the uncertainty estimated by considering SU(3) symmetry breaking. We also estimated the sensitivities of the measurements of these rare decays at the future experiments, such as BES-III, super-$B$ and LHC-$b$. Especially, the decays of $D_S^+ \to D^0 e^+ \nu$ and $B_S^0 \to B^+ e^- \bar{\nu}$ may be reached at the super-B factory. Observations of these decays will be used to test the SM predictions for the rare simleptonic decays. Further more, any indication of deviation from the SM prediction may shed light on the searches of New Physics.

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